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A short note on some properties of rough groups

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ABSTRACT

It is a useful method in research of group theory to construct a new group by using known groups. Lower and upper approximation operators of rough sets are applied into group theory and so the notion of a rough group has been introduced. In this paper, we first point out that there are still some incomplete propositions in [N. Kuroki, P.P. Wang, The lower and upper approximations in a fuzzy group, Inform. Sci. 90 (1996) 203–220] although some authors have showed several incorrect statements in the literature. We then present improved versions of the incomplete propositions and continue to study the image and inverse image of rough approximations of a subgroup with respect to a homomorphism between two groups.

1. Introduction

The theory of rough sets, proposed by Pawlak [1], is an excellent mathematic tool to deal with granularity of information. The indiscernibility relation is the mathematical basis for rough set theory. The indiscernible objects form an elementary set and all elementary sets form a partition of the universe. Any subset of the universe, being a union of some elementary sets, is called a definable set. Otherwise, it is called a rough set. Therefore, every rough set cannot be characterized by elementary sets precisely. Instead, a rough set could be roughly described by a pair of crisp sets, called the lower and the upper approximations. The main idea of rough set theory is to deal with classification and to extract decision rules from a decision system. By using the concepts of lower and upper approximations in rough set theory, knowledge hidden in information systems may be unraveled and expressed in the form of decision rules [2–6,1,7–16].

There are mainly two approaches for the development of rough set theory, the constructive and axiomatic approaches. By taking advantage of these two approaches, rough set theory has been combined with other mathematical theories such as modal logic [17], Boolean algebra [18,19], fuzzy sets [20–22,3,23,10,24], semigroup [25–27], and random set [11,28]. Among these research aspects, many papers has been focused on the connection between rough sets and algebraic systems. Biswas and Nanda [29] defined the notion of rough subgroups. Kuroki [26] introduced the notion of a rough ideal in a semigroup, studied approximations of a subset in a semigroup and discussed the product structures of a rough ideal. In [30], Kuroki and Wang provided some propositions in an investigation of the properties of lower and upper approximations with respect to normal subgroups. However, there are still some of these statements which appear to be incomplete, although Cheng [31] have showed that some propositions in [30] are incorrect. Davvaz [32] examined a relationship between rough sets and ring theory and introduced the notions of rough ideals and rough subrings with respect to an ideal of a ring. In [33], Davvaz and Mahdavipour considered an R-module as a universal set and introduced the notion of rough submodule with respect to a submodule of an R-module. In [34], Kazanc and Davvaz further introduced the notions of rough prime (primary) ideals and rough fuzzy prime (primary) ideals in a ring and presented some properties of such ideals. In [27], the notions of rough prime ideals and rough fuzzy prime ideals in a semigroup were introduced. In [35], Jiang and Chen studied the product structures

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of rough fuzzy sets on a group and proposed the notions of T-rough fuzzy subgroups in a group with respect to a T-fuzzy normal subgroup.

As discussed above, lower and upper approximation operators of rough sets are applied into group theory and so the notion of a rough group has been introduced. In this paper, we point out that there are still some incomplete propositions in [30] although some authors [31] have showed several incorrect statements in the literature. We then present the modified versions of them and continue to study the image and inverse image of rough approximations of a subgroup with respect to a homomorphism between two groups and prove that the lower and upper approximations of the image and inverse image of a subgroup are equal to the image and inverse image of the lower and upper approximations of the subgroup with respect to a group homomorphism, respectively.

This paper is structured as follows. In Section 2 we review some basic notions of rough groups and some main results about approximate structures of rough groups. In Section 3 we show that some of the propositions in [30] are incomplete and improve them. In Section 4 we study homomorphic images of rough approximations of a subgroup. Finally, our conclusions are presented.

2. Preliminaries

In this section, we review some basic notions of rough groups and some main statements to be used in the following sections [31,30].

Definition 2.1. Let G be a group with identity e. Let R be an equivalence relation on G. Then R is called a congruence relation of G if R satisfies the following condition:

$$\forall x \in G, \quad (a, b) \in R \Rightarrow (ax, bx), (xa, xb) \in R.$$

Definition 2.2. Let N be a normal subgroup of G and A a nonempty subset of G. Let

$$N_{-}(A) = \{x \in G : xN \subseteq A\},$$
$$N^{-}(A) = \{x \in G : xN \cap A \neq \emptyset\}.$$

Then $N_{-}(A)$ and $N^{-}(A)$ are called lower and upper approximations of A with respect to the normal subgroup N, respectively.

Theorem 2.3. *Let N and H be normal subgroups of a group G. Let A and B be two nonempty subsets of G. Then*
(1) $N^{-}(A)N^{-}(B) = N^{-}(AB)$, $N_{-}(A)N_{-}(B) \subseteq N_{-}(AB)$;
(2) $(H \cap N)^{-}(A) \supseteq H^{-}(A) \cap N^{-}(A)$, $(H \cap N)_{-}(A) \subseteq H_{-}(A) \cap N_{-}(A)$.
We can define new algebraic structures by using the concepts of lower and upper approximations and congruence relations.

Definition 2.4. Let N be a normal subgroup of a group G and A a nonempty subset of G. Then A is called a upper rough subgroup (respectively, normal subgroup) of G if $N^{-}(A)$ is a subgroup (respectively, normal subgroup) of G. Similarly, A is called a lower rough subgroup (respectively, normal subgroup) of G if $N_{-}(A)$ is a subgroup (respectively, normal subgroup) of G.

Theorem 2.5. *Let N be a normal subgroup of a group G and A a subgroup (respectively, a normal subgroup) of G. Then A is a upper rough subgroup (respectively, normal subgroup) of G.*

Theorem 2.6. *Let N be a normal subgroup of a group G, A a subgroup (respectively, normal subgroup) of G, and $N \subseteq A$. Then A is a lower rough subgroup (respectively, normal subgroup) of G.*

3. Notes on some properties of rough groups

In this section, we indicate that some propositions in [30] are incomplete and give modified versions of them.

Proposition 3.1 (*Kuroki and Wang [30, p. 208 Proposition 3.5]*). *Let H and N be normal subgroups of a group G. If A is a subgroup of G, then*

$$H^{-}(A)N^{-}(A) \subseteq (HN)^{-}(A).$$

We give the modified version of the above proposition in Proposition 3.2.

Proposition 3.2. *Let H and N be normal subgroups of a group G. If A is a subgroup of G, then*

$$H^{-}(A)N^{-}(A) = (HN)^{-}(A).$$

Proof. The proof of the proposition $H^{-}(A)N^{-}(A) \subseteq (HN)^{-}(A)$ is presented in [30]. Next, we only need to prove that $H^{-}(A)N^{-}(A) \supseteq (HN)^{-}(A)$.

Let x be any element in $(NH)^{-}(A)$; it follows from the definition of $(NH)^{-}(A)$ that $x(HN) \cap A \neq \emptyset$, which implies that there exists $y \in G$ such that $y \in x(HN) \cap A$ and so $y \in x(HN)$ and $y \in A$. Hence there exist $a \in H$, $b \in N$ such that $y = xab$.

Since $y = xab \in xaN$ and $y \in A$, we have $xaN \cap A \neq \emptyset$, which implies $xa \in N^-(A)$. Thus $x \in N^-(A)a^{-1}$. Since $a \in H \subseteq H^-(A)$, it follows from Definition 2.4 and Theorem 2.5 that $x \in N^-(A)H^-(A)$. Hence $H^-(A)N^-(A) \supseteq (HN)^-(A)$. □

Proposition 3.3 (*Kuroki and Wang [30, p. 209 Proposition 3.6]*). *Let H and N be normal subgroups of a group G. If A is a subgroup of G, then*

$$(HN)^-(A) \subseteq H^-(A)N \cap N^-(A)H.$$

We give the modified version of the above proposition in Proposition 3.4.

Proposition 3.4. *Let H and N be normal subgroups of a group G. If A is a subgroup of G, then*

$$(HN)^-(A) = H^-(A)N \cap N^-(A)H.$$

Proof. The proof of the proposition $(HN)^-(A) \subseteq H^-(A)N \cap N^-(A)H$ is presented in [30]. Next, we only need to give the proof of the inverse inclusion.

Let x be any element in $H^-(A)N$; then there must exist y and $z \in G$ such that $y \in H^-(A)$, $z \in N$ and $x = yz$. Thus $yH \cap A \neq \emptyset$, which implies that there exists $a \in G$ such that $a \in yH$ and $a \in A$. Hence $y \in aH$ and $yz \in aHN$. Since H and N are normal subgroups of G, then HN is also a normal subgroup of G. So we get $a \in yz(HN)$. This implies that $yz(HN) \cap A \neq \emptyset$. Thus $yz \in HN^-(A)$, i.e., $x \in (HN)^-(A)$. Hence $H^-(A)N \subseteq (HN)^-(A)$. Similarly, $N^-(A)H \subseteq (HN)^-(A)$. Therefore, $(HN)^-(A) \supseteq H^-(A)N \cap N^-(A)H$. □

Proposition 3.5 (*Kuroki and Wang [30, p. 209 Proposition 3.7]*). *Let H and N be normal subgroups of a group G. If A is a subgroup of G, then*

$$H_-(A)N_-(A) \subseteq (HN)_-(A).$$

Let $\emptyset$ be an empty set and A a non-empty subset of a group G. Let us make an appointment: the product of $\emptyset$ and A is defined as follows:

$$\emptyset A = \emptyset \quad \text{and} \quad A\emptyset = \emptyset.$$

Now, we give the modified version of the above proposition as follows.

Proposition 3.6. *Let H and N be normal subgroups of a group G. If A is a subgroup of G, then*

$$H_-(A)N_-(A) = (HN)_-(A).$$

Proof. First, we are to prove the fact that, if $H_-(A) \neq \emptyset$, then $H_-(A) = A$.

For any $x \in H_-(A)$, by the definition of $H_-(A)$ we have $xH \subseteq A$. Thus $x = xe \in A$, where e is the identity element of H. Since A is a subgroup of G, it follows that $x^{-1} \in A$. Hence $H = x^{-1}xH \subseteq AA \subseteq A$.

It is obvious that $H_-(A) \subseteq A$ is true. We only need to prove $A \subseteq H_-(A)$.

Let $a \in A$. By $H \subseteq A$, we have $aH \subseteq AA \subseteq A$ which implies $a \in H_-(A)$. Thus $A \subseteq H_-(A)$. So we get $H_-(A) = A$. Similarly, we have the fact that $N_-(A) \neq \emptyset$, then $N_-(A) = A$.

If $H_-(A) \neq \emptyset$ and $N_-(A) \neq \emptyset$, by the above proof we have $H_-(A) = A$ and $N_-(A) = A$. Then $H_-(A)$ and $N_-(A)$ are subgroups of G. This means that the identity element $e \in H_-(A)$ and $e \in N_-(A)$. So we have $H \subseteq A$ and $N \subseteq A$, and so $HN \subseteq A$. Hence $e \in (HN)_-(A) \neq \emptyset$. Similar to the proof of $H_-(A) = A$, we have $(HN)_-(A) = A$.

Now we are to prove that, if $H_-(A) = \emptyset$, then $(HN)_-(A) = \emptyset$.

Since $H_-(A) = \emptyset$, we have $aH \subsetneq A$ for any $a \in G$, which implies $aH \subsetneq NA$. Thus $a \notin (HN)_-(A)$. Hence $(HN)_-(A) = \emptyset$. This completes the proof. □

Theorem 3.1. *Let N be a normal subgroup of a group G and A a nonempty subset of G. Then* $N^-(A) = AN$.

Proof. Let x be any element of $N^-(A)$; then $xN \cap A \neq \emptyset$. This means that there exists $y \in G$ such that $y \in xN$ and $y \in A$. Since N is a normal subgroup of G, then $x \in yN$ and $y \in A$, and so $x \in AN$. Hence $N^-(A) \subseteq AN$.

On the other hand, let x be any element of AN; then there exist $a \in A$ and $n \in N$ such that $x = an$. Thus $a = xn^{-1}$. By the fact that N is a normal subgroup of G, we have $a = xn^{-1} \in xN$, which implies $a \in xN \cap A \neq \emptyset$. Thus $x \in N^-(A)$. Hence $AN \subseteq N^-(A)$. Therefore, we have $N^-(A) = AN$. □

Theorem 3.2. *Let N be a normal subgroup of a group G and A a subgroup of G. If* $N \subsetneq A$*, then* $N_-(A) = \emptyset$*; if* $N \subseteq A$*, then* $N_-(A) = A$.

Proof. First, we are to prove the fact that, if $N \subsetneq A$, then $xN \subsetneq A$ for any $x \in G$.

Assume that there exists $x \in G$ such that $xN \subseteq A$; then $x = xe \in A$, where $e \in N$ is the identity element. Since A is a subgroup of G, it follows that $x^{-1} \in A$. Thus

$$x^{-1}xN \subseteq AA \subseteq A.$$

That is, $N \subseteq A$. This is a contradiction. Hence, $\forall x \in G$, $xN \subsetneq A$. So we get $N_-(A) = \emptyset$.

If $N \subseteq A$, then $xN \subseteq AA \subseteq A$ for any $x \in A$. This implies $x \in N_-(A)$. Thus $A \subseteq N_-(A)$. Hence $N_-(A) = A$. □

From Theorem 3.2 we can see that, if a given nonempty subset A is a subgroup of a group G, then $N_-(A) = \emptyset$ or $N_-(A) = A$. In the following we apply Theorems 3.1 and 3.2 to present another method for proof of the improved versions of the incomplete propositions in [30].

The proof of Proposition 3.2 by Theorem 3.1 is as follows.

Proof. Since H and N are normal subgroups of G and A is a subgroup of G, by Theorem 3.1 we have $H^-(A) = AH$ and $N^-(A) = AN$. Thus $H^-(A)N^-(A) = AH \cdot AN$, which implies $H^-(A)N^-(A) = AAHN = AHN = (HN)^-(A)$. □

The proof of Proposition 3.4 by Theorem 3.1 is as follows.

Proof. Since H and N are normal subgroups of G and A is a subgroup of G, by Theorem 3.1 we have $H^-(A) = AH$ and $N^-(A) = AN$. Thus $H^-(A)N = AHN$ and $N^-(A)H = ANH = AHN$. Hence $H^-(A)N \cap N^-(A)H = AHN = (HN)^-(A)$. □

The proof of Proposition 3.6 by Theorem 3.2 is as follows.

Proof. Since H and N are normal subgroups of G and A is a subgroup of G, by Theorem 3.2 we have the following four cases:

If $N \subsetneq A$ and $H \subsetneq A$, then $HN \subsetneq A \cdot A \subseteq A$, $N_-(A) = \emptyset$, and $H_-(A) = \emptyset$. Thus $H_-(A)N_-(A) = \emptyset \cdot \emptyset = \emptyset = (HN)_-(A)$.

If $N \subsetneq A$ and $H \subseteq A$, then $HN \subsetneq A \cdot A \subseteq A$, $N_-(A) = \emptyset$, and $H_-(A) = A$. Thus $H_-(A)N_-(A) = A \cdot \emptyset = \emptyset = (HN)_-(A)$.

If $N \subseteq A$ and $H \subsetneq A$, then $HN \subsetneq A \cdot A \subseteq A$, $N_-(A) = A$, and $H_-(A) = \emptyset$. Thus $H_-(A)N_-(A) = \emptyset \cdot A = \emptyset = (HN)_-(A)$.

If $N \subseteq A$ and $H \subseteq A$, then $HN \subseteq A \cdot A \subseteq A$, $N_-(A) = A$, and $H_-(A) = A$. Thus $H_-(A)N_-(A) = A \cdot A = A = (HN)_-(A)$ by the fact that A is a subgroup of G. □

4. Homomorphic images of rough approximations

In this section we mainly study the image and inverse image of rough approximations of a subgroup with respect to a homomorphism between two groups. It is proven that the lower and upper approximations of a subgroup are invariant under a group homomorphism.

Theorem 4.1. *Let N and H be normal subgroups of a group G. Then*

(1) $(G/N)/(N^-(H)/N) \cong (G/H)/(H^-(N)/H)$;

(2) *If* $N \subseteq H$, *then* $G/N_-(H) \cong (G/N)/(N_-(H)/N)$.

Proof. (1) Since N and H are normal subgroups of G, by Theorem 2.5 it follows that $N^-(H)$ and $H^-(N)$ are normal subgroups of G, respectively. By the Second Isomorphism Theorem we have

$$G/N^-(H) \cong (G/N)/(N^-(H)/N) \quad \text{and} \quad G/H^-(N) \cong (G/H)/(H^-(N)/H).$$

It follows from Theorem 3.1 that $N^-(H) = H^-(N)$. Hence

$$(G/N)/(N^-(H)/N) \cong (G/H)/(H^-(N)/H).$$

(2) Follows directly from Theorem 3.2, and the Second Isomorphism Theorem. □

Let G_1 and G_2 be two groups. A mapping $f : G_1 \to G_2$ is called a homomorphism from G_1 to G_2 provided $f(ab) = f(a)f(b)$ for all $a, b \in G_1$. If f is surjective, f is called an epimorphism. Suppose $f : G_1 \to G_2$ is a homomorphism, and A and A' are subgroups of G_1 and G_2, respectively. We know that $f(A)$ and $f^{-1}(A')$ are subgroups of G_1 and G_2, respectively. If A and A' are normal, then $f(A)$ and $f^{-1}(A')$ are also normal.

Theorem 4.2. *Let G_1 and G_2 be two groups. Let f be an epimorphism from G_1 to G_2, N a normal subgroup of G_1 and A a subgroup of G_1. Then*

(1) $f(\underline{R_N}(A)) = \underline{R_{f(N)}}(f(A))$,

(2) $f(\overline{R_N}(A)) = \overline{R_{f(N)}}(f(A))$.

Proof. (1) Since f is an epimorphism from G_1 to G_2, N a normal subgroup of G_1 and A a subgroup of G_1, then $f(N)$ and $f(A)$ are a normal and a subgroup of G_2 respectively. Since $N \subseteq A \Leftrightarrow f(N) \subseteq f(A)$, by Theorem 3.2 we can get $\underline{R_N}(A) = A \Leftrightarrow \underline{R_{f(N)}}(f(A)) = f(A)$. Thus $f(\underline{R_N}(A)) = \underline{R_{f(N)}}(f(A))$.

(2) For any element $a \in \overline{R_N}(A)$, by the definition of $\overline{R_N}(A)$ we have $aN \cap A \neq \emptyset$.

Since $f(aN \cap A) \subseteq f(aN) \cap f(A)$, it follows that $f(aN) \cap f(A) \neq \emptyset$, which implies $f(a)f(N) \cap f(A) \neq \emptyset$. Thus $f(a) \in \overline{f(N)}(f(A))$. Hence $f(\overline{R_N}(A)) \subseteq \overline{f(N)}(f(A))$.

On the other hand, for any element $y \in \overline{R_{f(N)}}(f(A))$, by the definition of $\overline{R_{f(N)}}(f(A))$ we have $yf(N) \cap f(A) \neq \emptyset$. Thus there exist $n \in N$ and $a \in A$ such that $yf(n) = f(a)$. Since N is a normal subgroup of G_1, then $f(N)$ is also a normal subgroup of G_2. Thus $(f(n))^{-1} \in f(N)$. Hence $y = f(a)(f(n))^{-1} = f(a)f(n^{-1}) = f(an^{-1})$. Since $a = \left(an^{-1}\right) n \in (an^{-1})N \cap A \neq \emptyset$, it follows that $an^{-1} \in \overline{R_N}(A)$. This means $y \in f(\overline{R_N}(A))$. Hence $\overline{f(N)}(f(A)) \subseteq f(\overline{R_N}(A))$. So we get $f(\overline{R_N}(A)) = \overline{f(N)}(f(A))$. □

Theorem 4.3. *Let G_1 and G_2 be two groups. Let f be an epimorphism from G_1 to G_2, N' a normal subgroup of G_2 and A' a nonempty subset of G_2. Then*

(1) $f^{-1}(\underline{R'_N}(A')) = \underline{R_{f^{-1}(N')}}(f^{-1}(A'))$,

(2) $f^{-1}(\overline{R'_N}(A')) = \overline{R_{f^{-1}(N')}}(f^{-1}(A'))$.

Proof. Similar to the proof of Theorem 4.2. □

Theorems 4.2 and 4.3 show that the lower and upper approximations of the image and inverse image of a subgroup are equal to the image and inverse image of the lower and upper approximations of the subgroup with respect to a group homomorphism, respectively.

Theorem 4.4. *Let G_1 and G_2 be two groups. Let f be an epimorphism from G_1 to G_2 and N a normal subgroup of G_1. Let A be any normal subgroup of G_1. Then*
(1) If $ker\ f \subseteq N$, then $G_1/\overline{R_N}(A) \cong G_2/\overline{R_{f(N)}}(f(A))$;
(2) If $ker\ f \subseteq N \subseteq A$, then $G_1/\underline{R_N}(A) \cong G_2/\underline{R_{f(N)}}(f(A))$.

Proof. (1) By Theorem 4.2, we get $f(\overline{R_N}(A)) = \overline{R_{f(N)}}(f(A))$. Since the complete inverse image of $f(\overline{R_N}(A))$ is $f^{-1}(f(\overline{R_N}(A))) = \overline{R_N}(A) \cdot ker\ f$, we only need to prove $\overline{R_N}(A) \cdot ker\ f = \overline{R_N}(A)$.

Since N and A are normal subgroups of G_1, it follows from Corollary 3.1 that $\overline{R_N}(A)$ is a normal subgroup of G_1. This means the identity element $e \in \overline{R_N}(A)$. Thus $eN \cap A \neq \emptyset$, which implies $N \cap A \neq \emptyset$. Hence we have $nN \cap A = N \cap A \neq \emptyset$ for any $n \in N$. This means $n \in \overline{R_N}(A)$. So $N \subseteq \overline{R_N}(A)$. Since $ker\ f \subseteq N$, then $ker\ f \subseteq N \subseteq \overline{R_N}(A)$. Thus $\overline{R_N}(A) \cdot ker\ f = \overline{R_N}(A)$. By the First Isomorphism Theorem we have $G_1/\overline{R_N}(A) \cong G_2/\overline{R_{f(N)}}(f(A))$.

(2) Since $ker\ f \subseteq N \subseteq A$, it follows from Theorem 3.2 that $\underline{R_N}(A) = A$ and $\underline{R_{f(N)}}(f(A)) = f(A)$. Since the complete inverse image of $f(A)$ is $f^{-1}(f(A)) = A \cdot ker\ f = A$, by the First Isomorphism Theorem we have $G_1/\underline{R_N}(A) \cong G_2/\underline{R_{f(N)}}(f(A))$. □

5. Conclusions

Rough set theory is important in both branches of mathematics: pure and applied. The study of properties of rough sets on a group is a meaningful research topic of rough set theory. The main objective of this paper is to point out some incomplete statements in the literature [30] and to contribute to making it more complete. We also discuss the homomorphic images of rough approximations of subgroups in a group. We hope that this extended research may provide a complete complement to some incomplete results in [30].

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